APPLICATION NOTE

Equations for calculation of the errors of anisotropy factors

First line represents equation for given anisotropy factor, where p_x is no. of given anisotropy factor (red letters represents usual abbreviation of the factor), $k_{1,2,3}$ are principal susceptibilities and $k = \frac{k_1+k_2+k_3}{3}$ is mean susceptibility . $n_{1,2,3}$ are natural logarithms of principal susceptibilities and n is natural logarithm of the mean susceptibility.

Parameters *a,b,c,d* express errors of the principal or mean susceptibilities as follows:
$a = \delta k_1, b = \delta k_2, c = \delta k_3, d = \delta k = \frac{1}{3}\sqrt{a^2+b^2+c^2}$

Parameters *A,B,C,D* express errors of the natural logarithms of principal or mean susceptibilities as follows:
$A = |\frac{\delta k_1}{k_1}|, B = |\frac{\delta k_2}{k_2}|, C = |\frac{\delta k_3}{k_3}|, D = |\frac{\delta k}{k}|$

δp_x express equation for the error of given anisotropy factor.

$$p_1 = \frac{15}{2}\frac{(k_1-k)^2+(k_2-k)^2+(k_3-k)^2}{(3k)^2}$$

$$\delta p_1 = \frac{5}{3}\sqrt{d^2\,k^6\left({k_3}^2+{k_2}^2+{k_1}^2-k\,\left(k_3+k_2+k_1\right)\right)^2+\frac{c^2\left(k_3-k\right)^2+b^2\left(k_2-k\right)^2+a^2\left(k_1-k\right)^2}{k^4}}$$

$$P' = p_2 = e^{\sqrt{2((n_1-n)^2+(n_2-n)^2+(n_3-n)^2)}}$$

$$\delta p_2 = \sqrt{\frac{2(D^2(3n-n_3-n_2-n_1)^2+C^2(n_3-n)^2+B^2(n_2-n)^2+A^2(n_1-n)^2)}{\left(n_3-n\right)^2+\left(n_2-n\right)^2+\left(n_1-n\right)^2}}\;\cdot$$
$$\cdot\, e^{\sqrt{2((n_3-n)^2+(n_2-n)^2+(n_1-n)^2)}}$$

$$ln(P') = p_3 = \sqrt{2[(n_1 - n)^2 + (n_2 - n)^2 + (n_3 - n)^2]}$$

$$\delta p_3 = \sqrt{\frac{2(D^2(3n - n_3 - n_2 - n_1)^2 + C^2(n_3 - n)^2 + B^2(n_2 - n)^2 + A^2(n_1 - n)^2)}{(n_3 - n)^2 + (n_2 - n)^2 + (n_1 - n)^2}}$$

$$P = p_4 = \frac{k_1}{k_3}$$

$$\delta p_4 = \frac{\sqrt{a^2\, {k_3}^2 + c^2\, {k_1}^2}}{{k_3}^2}$$

$$ln(P) = p_5 = ln\frac{k_1}{k_3}$$

$$\delta p_5 = \sqrt{\frac{c^2}{{k_3}^2} + \frac{a^2}{{k_1}^2}}$$

$$p_6 = 100\frac{k_1 - k_3}{k_1}$$

$$\delta p_6 = \frac{100\sqrt{a^2\, {k_3}^2 + c^2\, {k_1}^2}}{{k_1}^2}$$

$$p_7 = \frac{k_1 - k_3}{k_2}$$

$$\delta p_7 = \frac{\sqrt{b^2\,(k_1 - k_3)^2 + c^2\,{k_2}^2 + a^2\,{k_2}^2}}{{k_2}^2}$$

$$p_8 = \frac{k_1 - k_3}{k}$$

$$\delta p_8 = \frac{\sqrt{d^2\,(k_1 - k_3)^2 + c^2\,k^2 + a^2\,k^2}}{k^2}$$

$$L = p_9 = \frac{k_1}{k_2}$$

$$\delta p_9 = \frac{\sqrt{a^2\,{k_2}^2 + b^2\,{k_1}^2}}{{k_2}^2}$$

$$ln(L) = p_{10} = ln\frac{k_1}{k_2}$$

$$\delta p_{10} = \sqrt{\frac{b^2}{{k_2}^2} + \frac{a^2}{{k_1}^2}}$$

$$p_{11} = \frac{k_1 - k_2}{k}$$

$$\delta p_{11} = \frac{\sqrt{d^2\,(k_1 - k_2)^2 + (b^2 + a^2)\,k^2}}{k^2}$$

$$p_{12} = \frac{2k_1}{k_2 + k_3}$$

$$\delta p_{12} = \frac{2\sqrt{a^2\,(k_3 + k_2)^2 + c^2\,{k_1}^2 + b^2\,{k_1}^2}}{(k_3 + k_2)^2}$$

$$F = p_{13} = \frac{k_2}{k_3}$$

$$\delta p_{13} = \frac{\sqrt{b^2\,{k_3}^2 + c^2\,{k_2}^2}}{{k_3}^2}$$

$$ln(F) = p_{14} = ln\frac{k_2}{k_3}$$

$$\delta p_{14} = \sqrt{\frac{c^2}{{k_3}^2} + \frac{b^2}{{k_2}^2}}$$

$$p_{15} = \frac{k_1 + k_2}{2k_3}$$

$$\delta p_{15} = \frac{\sqrt{(b^2 + a^2)\;{k_3}^2 + c^2\,(k_2 + k_1)^2}}{2{k_3}^2}$$

$$p_{16} = \frac{k_1 + k_3}{2k_2}$$

$$\delta p_{16} = \frac{\sqrt{b^2\,(k_3 + k_1)^2 + (c^2 + a^2)\;{k_2}^2}}{2{k_2}^2}$$

$$p_{17} = \frac{2k_2}{k_1 + k_3}$$
$$\delta p_{17} = \frac{2\sqrt{b^2 \left(k_3 + k_1\right)^2 + c^2 \, {k_2}^2 + a^2 \, {k_2}^2}}{\left(k_3 + k_1\right)^2}$$

$$p_{18} = \frac{1 - k_3}{k_2}$$
$$\delta p_{18} = \frac{\sqrt{a^2 \, {k_2}^2 + b^2 \left(1 - k_1\right)^2}}{{k_2}^2}$$

$$p_{19} = \frac{2k_1 - k_2 - k_3}{k_1 - k_3}$$
$$\delta p_{19} = \frac{\sqrt{\left(b^2 + a^2\right) \, {k_3}^2 + \left(-2a^2 \, k_2 - 2b^2 \, k_1\right) k_3 + \left(c^2 + a^2\right) \, {k_2}^2 - 2c^2 \, k_1 k_2 + \left(c^2 + b^2\right) \, {k_1}^2}}{\left(k_3 - k_1\right)^2}$$

$$p_{20} = \frac{(k_1 + k_2)/2 - k_3}{k}$$
$$\delta p_{20} = \frac{\sqrt{\left(b^2 + a^2\right) \, {k_3}^2 + \left(-2a^2 \, k_2 - 2b^2 \, k_1\right) k_3 + \left(c^2 + a^2\right) \, {k_2}^2 - 2c^2 \, k_1 k_2 + \left(c^2 + b^2\right) \, {k_1}^2}}{2\left(k_3 - k_1\right)^2}$$

$$p_{21} = \frac{k_2 - k_3}{k}$$
$$\delta p_{21} = \frac{\sqrt{d^2 \left(k_3 - k_2\right)^2 + \left(c^2 + b^2\right) \, k^2}}{k^2}$$

$$p_{22} = \frac{k_1}{\sqrt{k_2 k_3}}$$

$$\delta p_{22} = \sqrt{\frac{a^2}{k_2 k_3} + \frac{b^2\, {k_1}^2}{4{k_2}^3 k_3} + \frac{c^2\, {k_1}^2}{4k_2 {k_3}^3}}$$

$$p_{23} = \frac{k_1 k_3}{k_2^2}$$

$$\delta p_{23} = \frac{\sqrt{\left(a^2\, {k_2}^2 + 4b^2\, {k_1}^2\right) {k_3}^2 + c^2\, {k_1}^2 {k_2}^2}}{{k_2}^2\, |k_2|}$$

$$Q = p_{24} = \frac{k_1 - k_2}{(k_1 + k_2)/2 - k_3}$$

$$\delta p_{24} = \frac{4\sqrt{(b^2 + a^2)\ {k_3}^2 + (-2a^2\, k_2 - 2b^2\, k_1)\, k_3 + (c^2 + a^2)\ {k_2}^2 - 2c^2\, k_1 k_2 + (c^2 + b^2)\ {k_1}^2}}{(2k_3 - k_2 - k_1)^2}$$

$$p_{25} = \frac{k_1 - k_2}{k_2 - k_3}$$

$$\delta p_{25} = \frac{\sqrt{(b^2 + a^2)\ {k_3}^2 + (-2a^2\, k_2 - 2b^2\, k_1)\, k_3 + (c^2 + a^2)\ {k_2}^2 - 2c^2\, k_1 k_2 + (c^2 + b^2)\ {k_1}^2}}{(k_3 - k_2)^2}$$

$$p_{26} = \frac{k_2 - k_3}{k_1 - k_2}$$

$$\delta p_{26} = \frac{\sqrt{(b^2 + a^2)\ {k_3}^2 + (-2a^2\, k_2 - 2b^2\, k_1)\, k_3 + (c^2 + a^2)\ {k_2}^2 - 2c^2\, k_1 k_2 + (c^2 + b^2)\ {k_1}^2}}{(k_2 - k_1)^2}$$

$$p_{27} = arcsin\sqrt{\frac{k_2 - k_3}{k_1 - k_3}}$$

$$\delta p_{27} = \frac{\sqrt{\frac{(b^2+a^2)\,k_3{}^2+(-2a^2\,k_2-2b^2\,k_1)k_3+(c^2+a^2)\,k_2{}^2-2c^2\,k_1k_2+(c^2+b^2)\,k_1{}^2}{(k_2-k_1)(k_3-k_2)}}}{2\,|k_3 - k_1|}$$

$$E = p_{28} = \frac{k_2^2}{k_1 k_3}$$

$$\delta p_{28} = \frac{|k_2|\sqrt{\left(a^2\,k_2{}^2 + 4b^2\,k_1{}^2\right)k_3{}^2 + c^2\,k_1{}^2 k_2{}^2}}{k_1{}^2 k_3{}^2}$$

$$p_{29} = k_2\frac{k_1 - k_2}{k_1(k_2 - k_3)}$$

$$\delta p_{29} = \sqrt{\frac{c^2\,(k_1-k_2)^2 k_2{}^2}{k_1{}^2 (k_2-k_3)^4} + a^2\left(\frac{k_2}{k_1\,(k_2-k_3)} - \frac{(k_1-k_2)\,k_2}{k_1{}^2\,(k_2-k_3)}\right)^2 + b^2\left(-\frac{k_2}{k_1\,(k_2-k_3)} + \frac{k_1-k_2}{k_1\,(k_2-k_3)} - \frac{(k_1-k_2)\,k_2}{k_1 (k_2-k_3)^2}\right)^2}$$

$$p_{30} = \frac{k_2/k_3 - 1}{k_1/k_2 - 1}$$

$$\delta p_{30} = \sqrt{\frac{c^2\,k_2{}^2}{\left(\frac{k_1}{k_2}-1\right)^2 k_3{}^4} + b^2\left(\frac{k_1\left(\frac{k_2}{k_3}-1\right)}{\left(\frac{k_1}{k_2}-1\right)^2 k_2{}^2} + \frac{1}{\left(\frac{k_1}{k_2}-1\right)k_3}\right)^2 + \frac{a^2\left(\frac{k_2}{k_3}-1\right)^2}{\left(\frac{k_1}{k_2}-1\right)^4 k_2{}^2}}$$

$$T = p_{31} = \frac{2n_2 - n_1 - n_3}{n_1 - n_3}$$

$$\delta p_{31} = \frac{2\sqrt{(B^2 + A^2)\ {n_3}^2 + (-2A^2\,n_2 - 2B^2\,n_1)\,n_3 + (C^2 + A^2)\ {n_2}^2 - 2C^2\,n_1 n_2 + (C^2 + B^2)\ {n_1}^2}}{(n_3 - n_1)^2}$$

$$U = p_{32} = \frac{2k_2 - k_1 - k_3}{k_1 - k_3}$$

$$\delta p_{32} = \frac{2\sqrt{(b^2 + a^2)\ {k_3}^2 + (-2a^2\,k_2 - 2b^2\,k_1)\,k_3 + (c^2 + a^2)\ {k_2}^2 - 2c^2\,k_1 k_2 + (c^2 + b^2)\ {k_1}^2}}{(k_3 - k_1)^2}$$

$$p_{33} = \frac{k_1 + k_2 - 2k_3}{k_1 - k_2}$$

$$\delta p_{33} = \frac{2\sqrt{(b^2 + a^2)\ {k_3}^2 + (-2a^2\,k_2 - 2b^2\,k_1)\,k_3 + (c^2 + a^2)\ {k_2}^2 - 2c^2\,k_1 k_2 + (c^2 + b^2)\ {k_1}^2}}{(k_2 - k_1)^2}$$

$$R = p_{34} = \frac{\sqrt{((k_1-k)^2+(k_2-k)^2+(k_3-k)^2)/3}}{k}$$

$$\delta p_{34} = \left[d^2 \left(\frac{-2\left(k_3-k\right)-2\left(k_2-k\right)-2\left(k_1-k\right)}{2\sqrt{3}k\sqrt{\left(k_3-k\right)^2+\left(k_2-k\right)^2+\left(k_1-k\right)^2}} - \frac{\sqrt{\left(k_3-k\right)^2+\left(k_2-k\right)^2+\left(k_1-k\right)^2}}{\sqrt{3}\,k^2} \right)^2 \right.$$
$$+ \frac{c^2\left(k_3-k\right)^2}{3k^2\left(\left(k_3-k\right)^2+\left(k_2-k\right)^2+\left(k_1-k\right)^2\right)} + \frac{b^2\left(k_2-k\right)^2}{3k^2\left(\left(k_3-k\right)^2+\left(k_2-k\right)^2+\left(k_1-k\right)^2\right)}$$
$$\left. + \frac{a^2\left(k_1-k\right)^2}{3k^2\left(\left(k_3-k\right)^2+\left(k_2-k\right)^2+\left(k_1-k\right)^2\right)} \right]^{1/2}$$

$$p_{35} = (k_1k_2k_3)^{\frac{1}{3}}$$

$$\delta p_{35} = \frac{\sqrt{\frac{1}{k_1^{\frac{4}{3}}\,k_2^{\frac{4}{3}}\,k_3^{\frac{4}{3}}}}\sqrt{\left(a^2\,{k_2}^2+b^2\,{k_1}^2\right){k_3}^2+c^2\,{k_1}^2{k_2}^2}}{3}$$

$$p_{36} = \frac{k_3(k_1-k_2)}{k_1(k_2-k_3)}$$

$$\delta p_{36} = \frac{\sqrt{\left(a^2\,{k_2}^2+b^2\,{k_1}^2\right){k_3}^4+\left(-2a^2\,{k_2}^3-2b^2\,{k_1}^3\right){k_3}^3+\left(a^2\,{k_2}^4+b^2\,{k_1}^4\right){k_3}^2+c^2\,{k_1}^2{k_2}^4-2c^2\,{k_1}^3{k_2}^3+c^2\,{k_1}^4{k_2}^2}}{{k_1}^2(k_3-k_2)^2}$$

$$p_{37} = \frac{k_3(k_1 - k_2)}{k_2^2 - k_1 k_3}$$

$$\delta p_{37} = \left[c^2 \left(\frac{\frac{d}{dk_3} k_3 \left(k_1 - k_2\right)}{{k_2}^2 - k_1 k_3} + \frac{k_1 k_3 \left(k_1 - k_2\right)}{\left({k_2}^2 - k_1 k_3\right)^2} \right)^2 + b^2 \left(\frac{\frac{d}{dk_2} k_3 \left(k_1 - k_2\right)}{{k_2}^2 - k_1 k_3} - \frac{2 k_3 \left(k_1 - k_2\right) k_2}{\left({k_2}^2 - k_1 k_3\right)^2} \right)^2 \right.$$
$$\left. + a^2 \left(\frac{\frac{d}{dk_1} k_3 \left(k_1 - k_2\right)}{{k_2}^2 - k_1 k_3} + \frac{k_3 \left(k_1 - k_2\right) k_3}{\left({k_2}^2 - k_1 k_3\right)^2} \right)^2 \right]^{1/2}$$

$$p_{38} = \frac{(k_1 - k_2)(2k_1 - k_2 - k_3)}{(k_2 - k_3)(k_1 + k_2 - 2k_3)}$$

$$\delta p_{38} = \left[a^2 \left(\frac{-k_3 - k_2 + 2k_1}{\left(-2k_3 + k_2 + k_1\right)\left(k_2 - k_3\right)} - \frac{\left(k_1 - k_2\right)\left(-k_3 - k_2 + 2k_1\right)}{\left(-2k_3 + k_2 + k_1\right)^2 \left(k_2 - k_3\right)} + \frac{2\left(k_1 - k_2\right)}{\left(-2k_3 + k_2 + k_1\right)\left(k_2 - k_3\right)} \right)^2 \right.$$
$$+ b^2 \left(- \frac{-k_3 - k_2 + 2k_1}{\left(-2k_3 + k_2 + k_1\right)\left(k_2 - k_3\right)} - \frac{\left(k_1 - k_2\right)\left(-k_3 - k_2 + 2k_1\right)}{\left(-2k_3 + k_2 + k_1\right)^2 \left(k_2 - k_3\right)} - \frac{k_1 - k_2}{\left(-2k_3 + k_2 + k_1\right)\left(k_2 - k_3\right)} - \frac{\left(k_1 - k_2\right)\left(-k_3 - k_2 + 2k_1\right)}{\left(-2k_3 + k_2 + k_1\right)\left(k_2 - k_3\right)^2} \right)^2$$
$$\left. + c^2 \left(\frac{2\left(k_1 - k_2\right)\left(-k_3 - k_2 + 2k_1\right)}{\left(-2k_3 + k_2 + k_1\right)^2 \left(k_2 - k_3\right)} - \frac{k_1 - k_2}{\left(-2k_3 + k_2 + k_1\right)\left(k_2 - k_3\right)} + \frac{\left(k_1 - k_2\right)\left(-k_3 - k_2 + 2k_1\right)}{\left(-2k_3 + k_2 + k_1\right)\left(k_2 - k_3\right)^2} \right)^2 \right]^{1/2}$$

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